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Thoughts from the front line: Current issues in real-time graphics and areas where Geometric Algebra can help

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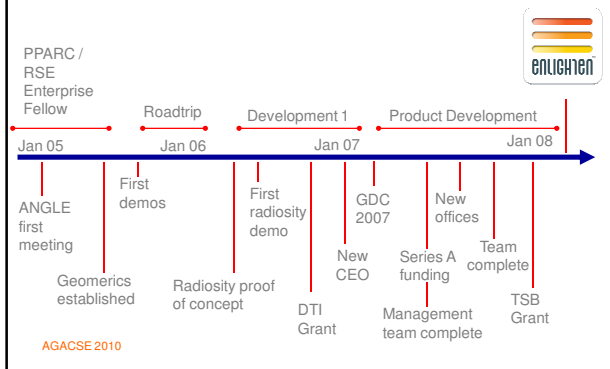
Introduction

- In 2005 I decided to take a break from academic research and formed Geomerics
- Looking for a new challenge
- Looking for other ways to win people over to GA



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Geomerics Timeline



What we do

- Enlighten is a revolutionary lighting technology available for PC, Xbox 360 and PS3
- Enlighten is being used in games by EA Dice (Battlefield), CCP, Funcom, all pushing for the highest quality lighting and graphics



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What we do



All images here are re-lit in real time at 30 fps, using Enlighten



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This talk

- Three areas which look interesting for future work on GA
 - Graphics
 - Discrete exterior calculus
 - Functional programming
- These are chosen not for their academic interest
- Areas where there is a real opportunity for GA to make an impact on a wider stage
- Also throwing in 3 puzzles / hobbyist topics

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Graphics

- An enormous topic, covered in depth in this conference
 - Radiosity
 - Global Illumination
 - Photon mapping
 - Ray tracing
 - Shadowing
 - Visibility
 - Ambient Occlusion
 - BRDF
 - Pre-computed radiance transfer

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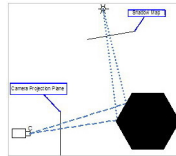
The biggest problems?

- For games, the key problems are:
- Shadowing dynamic objects from direct lights
 - Shadow maps ...
- Soft shadows from area lights
 - Convolution shadow maps ...
- Dynamic object radiosity
 - Dynamic ambient occlusion, screen-space techniques ...

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Shadows from Direct Lights

- A solved problem to some extent:
- Create a shadow map for each light source by rendering depth information
- Use this to look up whether or not a point is in shadow
- Gives rise to jaggies, aliasing artifacts ...



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Refinements

- Many ways to improve basic shadow maps
 - Deform the geometry so that the shadow map better reflects the camera orientation (paraboloidal SMs)
 - Introduce a 'cascade' of shadow maps to prevent horrible blocky shadows from distant sources
- But basic problems remain:
 - High quality results using shadow maps requires high resolution maps
 - These are slow and limit the number of direct light sources that can be used in real time
 - Not obvious how to filter

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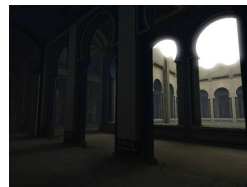
What we would like!

- A solution based on rendering from light sources that:
 - Massaged the geometry in a useful way before rendering
 - Stored more than just depth (a plane, point + line ...)
 - Ideally in a form that could be low resolution and amenable to filtering
 - Implemented as a simple screen-space step (potentially where filtering came in)
- Remember:
 - Will always trade off accuracy for speed
 - A nicely blurred approximate answer often works well

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Soft Shadows

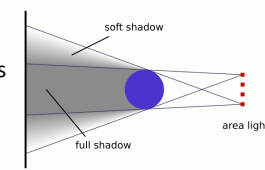
- Soft shadows are generated by area lights and are everywhere



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Soft Shadows

- This is a really hard problem!
- Can break into two aspects
 - Area lights
 - Full blown radiosity
- We have made good progress with radiosity
- But accurate area lights are unsolved for real-time graphics



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Area Lights

- The ability for an artist to dynamically place area lights with correct soft shadows would revolutionise work flow
- Any GA tricks?
 - Light sources as circles
 - Fractional / approximate visibility
 - Ability to blur simple shadows in an appropriate texture (see eg convolutions shadow maps)

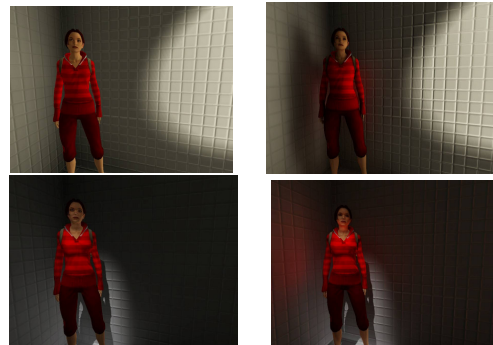
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Dynamic object radiosity

- In Enlighten we make a number of compromises:
 - Radiosity is computed for static geometry
 - Involves an off-line pre-compute
 - Light sources can move and change in real time
 - Dynamic objects are lit by the radiosity
 - Appear to be rooted in their world
 - But dynamic objects do not shadow the radiosity or bleed colour

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Dynamic object radiosity



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Dynamic Radiosity

- The big unsolved problem
- Need fast, approximate visibility updates
- Re-creation of form factors is less important
- Need to replace hierarchical data structures with something more malleable
- Incorporation of surface reflection properties
- Possibly screen-space type approach (caution!)
- Volume based or surface based?

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Interlude 1

- Occasional frustrations with conformal GA
- Often want to drop back to affine or projective framework
- Somehow this is never easy
- Elementary pieces of geometry turn into lengthy un-inspired algebra
- Consider same basic triangle results:

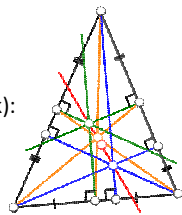
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Simple Triangle

- Circumcenter (green) is easy
- Centroid (orange) is (after some work):

$$G \wedge n = I(L_1 \times L_2 + L_2 \times L_3 + L_3 \times L_1)$$
- Tricky, but at least it is transparently symmetric
- Orthocentre (blue) is yet more difficult

$$H \wedge n = I \langle n L_1 L_2 L_3 \rangle_2$$
- Anyone got a simple proof of the Euler line?



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Discrete exterior calculus

$$\int_M d\omega = \int_{\partial M} \omega.$$

- Work of Desbrun, Marsden, Hirani and others
- An attempt to develop a formal discrete theory of differential forms
 - Every continuous concept has a discrete analog
- We MUST develop a GA version of this theory
 - Otherwise the graphics community will be lost to exterior geometry for good!

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Objects in DEC

- Discrete versions of each of
 - Differential forms
 - wedge product
 - Vector fields (and higher dimensions)
 - exterior derivative
 - Codifferential
 - Hodge star
 - Flat and sharp operators
 - Contraction
 - Lie derivative, Laplace – deRham operator, etc...

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Foundations of DEC

- All defined in such a way that the main theorems are automatically true
- All very reminiscent of Hestenes and Sobczyk's approach to the foundations of geometric calculus
- Chose your definitions carefully so that the key result is transparent

$$\oint L(dS) = \int L(\tilde{\nu} \cdot dX)$$

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Concepts in DEC

- 1-forms are numbers attached to edges
- 2-forms are numbers attached to planes
- And so on. All seems utterly obvious.
- But no useful notion of direction – the 1-form has to have the direction of the edge
- We need a notion of a vector field to discretise Maxwell equations (or anything else useful)
- At this point a dual manifold is introduced, based on either barycentric duals or centroids

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Dual Manifold

- The dual manifold is the first point where things go awry
 - Vector fields look un-natural
- The wedge product is quite horrific
- It takes pages to prove the main results of the product
 - They should be obvious by definition
- From then on it all feels like a struggle

$$\langle \alpha^k \wedge \beta^l, \sigma^{k+l} \rangle = \frac{1}{(k+l)!} \sum_{\tau \in S_{k+l}} \text{sign}(\tau) \frac{|\sigma^{k+l} \cap \star v_{\tau(k)}|}{|\sigma^{k+l}|} \alpha \sim \beta(\tau(\sigma^{k+l})),$$

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Hasn't this all been done?

- NO!
- Discrete exterior calculus is a recent development and actively ongoing
- Despite its difficulties it is comfortably the most complete and impressive theory we have
- With work, discrete analogs of most continuum results can be found
- We have no equivalent discrete theory within GA
 - This was not what Hestenes and Sobczyk were after

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Simple Example

- 2D vector derivative (aka the Cauchy-Riemann equations)

$$\nabla\psi = 0$$

- This is surprisingly hard to discretise
- Partly because the operator only propagates the part of the boundary data consistent with analyticity
- Can start from the Cauchy integral formula
- But then lose the ability to extend to curved surfaces
- And this is a problem of real practical significance!

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The right approach

- Some wild speculation:
 - The idea of defining scalars at points, 1-forms on lines, 2-forms on surfaces etc may not be the way to go
 - Instead, should we be defining a complete GA at discrete points?
 - Then need an operator for connecting adjacent algebras
 - This approach is more in the spirit of jet theory (see Olver: Equivalence, Invariants and Symmetry)
 - In jet theory differential equations are reduced to algebraic equations at a point, plus contact relations

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What is required

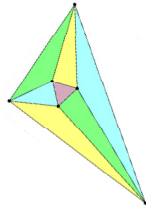
- A discrete vector manifold theory
- Based on the geometric product in the obvious way
- With a discrete vector derivative, and a discrete version of the fundamental theorem

$$\oint L(dS) = \int L(\tilde{\nabla} \cdot dX)$$
- The applications for such a theory would be vast
 - EMM, elasticity, re-meshing, numerical pdes ...
- This is the problem I would be focussing all efforts on!

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Interlude 2

- The Morley triangle, formed from angle tri-sectors
- Alain Connes has an algebraic proof of the result at www.alainconnes.org/docs/morley.pdf
- This proof involves
 - Complex projective geometry
 - Rotations from reflections
 - Fixed points of twists
- A conformal GA version please!



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Functional Programming

- Recently become interested in the functional programming language Haskell
- Will talk through its main features, and why it looks perfect for GA
- Functional languages are currently generating considerable interest:
 - Haskell, ML, ocaml ...
 - Microsoft developing F#, and supporting Haskell

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Haskell is a functional language

- Key objects are functions that take in arguments and return values (or functions)
- Mathematically this is simple, but far removed from modern object-oriented programming
- Means we give up on mutable objects
 - Never change a variable
 - Always create a new variable, then let garbage collector free up memory
- Focussing on functions gives compiler much better chance of parallelising code

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Haskell is a 'pure' language

- Pure functions have no I/O side effects
- Un-used results can be discarded
- Compiler can use tricks like memoization
- Evaluations are thread-safe
 - Good for parallelisation again
- Pure functional code can have various compiler optimisations applied
- In practice, Haskell code is mostly pure with a small amount of I/O

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Haskell is strongly typed

- Haskell contains a powerful type system
- Everything has a type
 - Functions map types to types, eg `Int -> Int`
- All code is checked for type integrity before compilation
- A lot of bugs are caught this way!
- Ties in with the concept that GA multivectors can remove ambiguity
 - Are 4 numbers are quaternion, a projective vector ...
 - Tracking blades removes all ambiguity

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Haskell has recursive functions

- In functional programming traditional for .. from ..to loops are replaced by other constructs
- Recursive functions are particularly useful

```
qsort [] = []
qsort (x:xs) = qsort (filter (< x) xs) ++ [x] ++ qsort (filter (>= x) xs)
```

- Use of recursion can shrink code dramatically
- Driving recursive definitions of functions is a powerful pattern matching framework
- Again, for mathematicians this is all natural!

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Haskell is a higher-order language

- Functions can take functions as arguments
- Functions can return functions as results
- Under the hood, functions are curried
 - Concept due to Haskell Curry
- All functions take in one parameter, and return a function / parameter
- Great for mapping functions to lists, etc

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Haskell is 'lazy'

- A defining property of Haskell is that function evaluation is lazy
- Functions are only evaluated when the result is needed elsewhere
 - Avoids unnecessary computation
 - Ensures programmes terminate where possible
 - Encourages good programming style
 - Allows for infinite lists
- Eg can define the 'infinite' list of all integers, and at a later date ask for the 10th element

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Haskell and GA

- This combination of properties makes Haskell uniquely suitable for GA
- Define blade and multivector data types


```
type GaBlade = (Float, GaBasis)
type GaMulti = [GaBlade]
```
- Says that a multivector is a list of blades
- Define a geometric product of blades, trivial to build up everything else
- Write code that mirrors hand-written algebra

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Laziness and GA

- Laziness is the key to Haskell's suitability

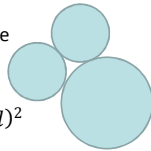
$$\langle AB \rangle_0$$
- Lazy evaluation ensures that only terms of grade zero are actually computed
- Can avoid vast amounts of hand optimisation this way
- Haskell will never be as fast as hand optimised C++ or intrinsics
- But it is far easier to write and debug, and promised much on multicore devices

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One final problem

- A fun problem from Martin Gardner's mathematical recreations
- Given three kissing circles:
 - Can always find two circles to kiss all three
- Inverse radii satisfy

$$a^2 + b^2 + c^2 + d^2 = \frac{1}{2}(a + b + c + d)^2$$
- A neat problem in conformal GA!



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Conclusions

- Many interesting open problems to explore with GA
- Opportunities to make a real difference in areas that will get GA widely noticed
 - Graphics, discrete theory, functional programming
- Plenty of drive from industry in setting the problem space, if people are interested
- And please come and talk to me if you make serious progress in any of these areas!

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